

Name: _____

- Factor $x^2 - 2x - 3$.
- Solve $x^2 - 2x - 3 = 0$ for x .
- Sketch a graph of $y = x^2 - 2x - 3$ – feel free to consult an electronic device or website.
- What are the x - and y -intercepts of that graph you just made? What do they have to do with the equation $y = x^2 - 2x - 3$?
- Multiply out $(x - (1 + 2i))(x - (1 - 2i))$. You should get a quadratic formula which has *only real numbers* (and the variable x). Call that formula $f(x)$.

6. Sketch a graph of $y = f(x)$ (using the $f(x)$ from problem 5). Again, electronic tools are OK.

7. What are the x - and y -intercepts of that graph you just made? What do they have to do with the equation $y = f(x)$?

8. Make a conjecture about how you can tell, by looking at a graph of a quadratic equation $y = ax^2 + bx + c$, whether it has real solutions or complex solutions? Come to think of it, usually we seem to have **two** real solutions, when we have any, for a quadratic equation. Can you refine your graphical conjecture to include the case of having **exactly one** real solution, too?

9. You solved $x^2 - 2x - 3 = 0$ above, by factoring. Now let's solve it by *Completing the Square*. First, can you find a (real) number a such that $(x + a)^2$ has the same x^2 and x terms as $x^2 - 2x - 3$? Tell what a is, and multiply out $(x + a)^2$, showing it matches at least the $x^2 - 2x$ we're working with.
10. Now find a (real) number b such that, using the a you just found, the $(x + a)^2 + b$ is exactly equal to $x^2 - 2x - 3$. (*I.e.g.*, adding b **fixes the constant** so that $(x + a)^2 + b = x^2 - 2x - 3$, with **all** the terms being equal, not just the x^2 and x terms.)
11. Now, use what you just found to solve $x^2 - 2x - 3 = 0$, as follows: Replace the equation $x^2 - 2x - 3 = 0$ by $(x + a)^2 + b = 0$ (using the a and b you just found) since $(x + a)^2 + b = x^2 - 2x - 3$. Solve $(x + a)^2 + b = 0$ by moving the b to the other side and taking the square root. Show/explain all of your work, clearly. [You should have gotten the same solutions as before!]

- 12.** Follow exactly the same process (of *completing the square*) to find the solution(s) to the equation $x^2 - 2x + 5 = 0$.