

Remember, what we care about is the *reasoning*, **not the final answer!** Explain all your work! Tell a story, let there be no characters (variables, axes in graphs, equations) without an individual introduction! Feel free to use a graphing tool like wolframalpha.com, desmos.com, or a graphing calculator.

First, a few facts and equations for exponents:

1. To start, if a is a positive real number and n is a positive whole number, then the definition of the exponent is

$$a^n = \overbrace{a \cdots a}^{n \text{ times}} .$$

2. Next, if a is any positive real number, and x and y are any two positive real numbers,

$$(a^x)^y = a^{x \cdot y} .$$

3. From that last one, we can see that if a is any positive real number and p and q are whole numbers with $q \neq 0$, then

$$a^{p/q} = \sqrt[q]{a^p} .$$

4. Also, if a is any positive real number, and x and y are any two positive real numbers,

$$a^x \cdot a^y = a^{x+y} .$$

5. What about negative powers, you ask? Well, if a and x are any two positive real numbers, then

$$a^{-x} = \frac{1}{a^x} .$$

6. Finally, if a is any positive real number (not 0), we have the simple expression

$$a^0 = 1 .$$

7. Now, if a is a real number such that $a > 1$, then the graph of $y = Ca^x$ has y -intercept C and increases (very rapidly). This is called **exponential growth** (and models things like growth of populations with unlimited resources, the amount of money in an interest-bearing bank account from which no withdrawals are made, *etc.*).
8. If, on the other hand, a is a real number such that $0 < a < 1$, then the graph of $y = Ca^x$ has y -intercept C , decreases (very rapidly), and is called **exponential decay** (and models things like radioactive decay and the real value of a constant number of dollars under inflation).

1. Simplify the following as much as you can, *showing/explaining all your work* [as always]. As you explain your work, if you use one of the above rules, say so, and tell which rule it was.

(a) 2^{-4}

(b) $(-2)^4$

(c) $\frac{1}{x^{-5}}$

(d) $x^9 \cdot x^{-7}$

(e) $\frac{x^4}{x^9}$

(f) $\frac{x^{-4}}{x^{-9}}$

(g) $(-2x)^4$

(h) $-(2x)^4$

2a. You start a bank account with 100\$. Each year, it earns 5% interest, which means: whatever is in the account at 11:59:59pm on December 31st, 5% of that amount is added to the account at 12:00:00 of January 1st. Suppose you are able to leave the money in this account and never take any out – it's just a long-term investment. Define variables and write a formula (or equation) which tells you how much is in your account as time goes by.

2b. Your friend starts a bank account with more money, but a worse interest rate: starting with 200\$, making 2% interest per year. Which of you will have more money in your account after 10, 20, 30, 40 years? Does it switch back and forth over time, which account has more?

- 3** Suppose a rumor – maybe a mean one, maybe just some previously unknown information – is spreading through the population of CSU-P students. Imagine that at first, only one person knows the rumor. Then imagine that, every hour after that, every person who knows the rumor tells one new person. Define variables and write an equation which models the spread of this rumor. How long until 50 people know the rumor? 100 people? The whole student body at CSU-P (about 5000 people)? All of the United States (population about 325 million people)? Everyone on Earth (7.3 billion people)?