

NAME _____

29 January 2009

MATH 124-002 — TEST I

PLEASE READ THESE INSTRUCTIONS:

Problems have varying point values, which are specified next to each problem number. Move quickly through the test, but try to be as accurate as you can. Do as many problems as you like; points over 100 will count as extra credit.

If you want credit for an answer, *you must explain what you are doing – this means defining your terms and explaining each step in your work.*

You may use a graphing calculator, but **not** a calculator (such as the *TI-89*) capable of symbolic manipulation. You may not use your book or notes.

PLEASE TURN OFF YOUR CELL PHONE.

[Please do not write in the boxes below:]

1 :	8
2 :	13
3 :	10
4 :	13
5 :	7
6 :	9
7 :	12
8 :	10
9 :	8
10 :	6
11 :	12
Σ :	108

(4+4) **1** What are the domains of the functions

$$\text{(a)} \quad f(x) = \frac{\sqrt{2+x}}{3-x} \quad \text{and} \quad \text{(b)} \quad g(x) = \sqrt{(6-3x)} \quad ?$$

(5+4+4) **2** Here is a graph of a function $g(x)$:

- (a) What are $g(-4)$, $g(-2)$, $g(0)$, $g(2)$ and $g(4)$?
- (b) Find the domain and range of g .
- (c) State the Vertical Line Test. What does it tell us about this graph?

(6+4) **3** Work with the graph in problem **2**, immediately above.

- (a) Specify two distinct intervals on which g is *decreasing* and one on which it is *increasing*.
- (b) What is the average rate of change of g from $x = -4$ to $x = 4$?

(6+7) 4 Graph the following functions:

$$g(x) = (x + 3)^2 - 1$$

$$h(x) = \begin{cases} -(x+1)^2 & \text{if } x < 0 \\ 4+x & \text{if } x \geq 0 \end{cases}$$

(2+5) 5 Start with the quadratic function $f(x) = 1 - x - x^2$.

- (a) Does $f(x)$ have a maximum or minimum? How do you know?
- (b) Find the vertex of $f(x)$: that is, the x and y coordinates of the extremum whose existence you just asserted – or, the *location* of the extremum and *its value*.

(2+3+4) 6 Given functions $f(x) = \sqrt{15+x}$ and $g(x) = x^2 + 1$, compute the following (simplify your answers!):

- (a) $(f \circ g)(3)$.
- (b) $(g \circ f)(x)$.
- (c) The domain of $(g \circ f)(x)$.

(4×3)

7

Here is the graph of a function $f(x)$:

Now sketch graphs of the following:

$$f(x - 2)$$

$$f(-x)$$

$$f(2x)$$

$$\frac{1}{2}f(x - 1)$$

(4+4+2) 8

Given the function $f(x) = \sqrt{x-5}$.

(a) What are the domain and range of $f(x)$? What are they for $f^{-1}(x)$?

(b) Sketch $f(x)$ and $f^{-1}(x)$ on the same axes (label the curves clearly).

(c) Find an expression for $f^{-1}(x)$.

(4+4) 9

Are the following functions **even**, **odd**, or **neither**? (Explain your reasoning!)

(a) $f(x) = x \cdot |x|$

(b) $f(x) = \frac{x}{x^2 + 1}$

- (6) **10** Find some functions f , g and h such that $f \circ g \circ h = F$, where $F(x) = \frac{1}{\sqrt[3]{x+1}}$.

- (12) **11** You have a 10 *cm* piece of wire. You cut it into two pieces and bend each of them into a square. How should you cut the wire so as to maximize the total area of these two squares? Or, if you prefer, figure out instead how you should cut the wire so as to minimize the total area of the two squares (but tell which version you are solving). *[Be sure to show your work completely and clearly.]*