

MATH 330 DO-AT-HOME MIDTERM II
SPRING TERM 2008

INSTRUCTIONS:

- This is an *open book and notes* test, so feel free to use your textbook and class notes. However, you must work on your own. Please also do not spend hours in the library (or on the Internet) looking for solutions to these problems.
- If you get completely stuck on a problem, feel free to contact me, however, and I could possibly consider a hint appropriate.
- Exposition is important: please be as clear as you can about what you are doing.
- This is **due in class on Wednesday, April 9th**.

PROBLEMS:

1. Do the following construction in **Geometry Explorer**:
 - a) Make a line segment $\overline{AB}$ of unit length (this can be *definition* of unit length in the construction).
 - b) Construct a line segment $\overline{AC}$, also of unit length, with $\overline{AC} \perp \overline{AB}$.
 - c) Construct a point D on $\overline{AC}$ which is $\frac{1}{8}$ units from C .
 - d) Construct a point E on $\overline{DB}$ such that $BE = \frac{1}{2}$.
 - e) Construct a point F on $\overline{AB}$ such that $\overline{EF} \perp \overline{AB}$.
 - f) Construct a point G on $\overline{AB}$ such that $\overline{EG} \parallel \overline{DF}$.
 - g) Construct a point H on the ray $\overrightarrow{AB}$ such that $BH = 1$.
 - h) Construct a point I on the ray $\overrightarrow{BH}$ such that $HI = 1$.
 - i) Construct a point J on the ray $\overrightarrow{HI}$ such that $IJ = GB$ and $HJ > 1$ (so J is past I on the ray).Now use **Geometry Explorer** to measure AJ . Do you recognize this value? Can you prove or disprove that this value is what you think it is? Derive a formula for the value of AJ by following along the construction and seeing what it implies about the lengths – that is, submit snapshots of the above construction, step by step, along with an explanation of each step in abstract geometric terms; also work with the geometry of the resulting figure (using similar triangles, the Pythagorean Theorem, *etc.*), to derive your formula for AJ .
2. Start with two congruent triangles $\triangle ABC \cong \triangle A'B'C'$.
 - a) Prove that there exists an Euclidean isometry which takes $\triangle ABC$ to $\triangle A'B'C'$, sending corresponding vertices to corresponding vertices.
 - b) What can you say about the isometry you have found if both $\triangle ABC$ and $\triangle A'B'C'$ are labeled in “clockwise order”?
 - c) How many isometries are there which take the set of points of the triangle $\triangle ABC$ to the set of points of triangle $\triangle A'B'C'$? Note your answer should depend on the geometric properties of $\triangle ABC$.
3. Start with two congruent triangles $\triangle ABC \cong \triangle A'B'C'$.
 - a) If both triangles are labeled in clockwise order, prove that the perpendicular bisectors of the segments $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$ either are parallel or meet at a single point.
 - b) If $\triangle ABC$ is labeled in clockwise order and $\triangle A'B'C'$ in counterclockwise order, prove that the midpoints of the segments $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$ all lie on a line.

[Hint: use the previous problem and what we know about compositions of different kinds of Euclidean isometries, and about fixed point sets of these different isometries.]