ETH Zürich, Dept. Informatik
WS 03/04

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Introduction to Quantum Computation
Übung 6

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6.1 Give an explicit example of a quantum operation coming from a unitary transformation on the total system/environment state space which does *not* act as a unitary transformation would on the density matric of the system alone – be very concrete here, with choices of Hilbert spaces and matrices.

6.2 Show that a pure state of a composite quantum system is entangled if and only if its reduction to a subsystem is mixed.

6.3 ([**NC**], Exercise 8.4): Suppose we have a single-qubit principal system interacting with a single-qubit environment through the tranform

$$U = P_0 \otimes I + P_1 \otimes X,$$

where X is the usual (Pauli) matrix and P_0 and P_1 are the projections on the computational coordinate axes (*e.g.*, $P_0 = |0\rangle\langle 0|$), and the first factor acts on the principal system. Give the quantum operation for this process, in the operator-sum representation, assuming the environment starts in the state $|0\rangle$. What does this have to do with measurement?

6.4 Describe in terms of quantum operations the quantum coin-tossing you invented to solve problem 2.8 from Übung 2.

6.5 Explain how to get from equation (8.13) to (8.14) in [**NC**].