ETH Zürich, Dept. Informatik
WS 03/04

Lecturer: Jonathan Poritz

Introduction to Quantum Computation
Übung 3

21.11.2002

3.1 Prove that $\{e^{2\pi i n\alpha} \mid n \in \mathbb{Z}\}$ is dense in the unit circle $\{z \in \mathbb{C} \mid |z| = 1\} \subset \mathbb{C}$ if and only if α is irrational.

3.2 [**NC**], problems 4.16–4.18 & 4.20

3.3 [**NC**], problems 4.33–4.35

3.4 **Unitarity of Permutations**

Let $n \in \mathbb{N}$ and σ be an arbitrary permutation of the set $\{0,1\}^n$ (i.e., σ is a bijective mapping defined on the bitstrings of length n). Let P_σ be the linear operation defined by

$$P_\sigma|b\rangle = |\sigma(b)\rangle$$

(for all $b \in \{0,1\}^n$).

Prove that P_σ is unitary (for any permutation σ).

3.5 **Permutations of Qubits**

Consider a circuit for n qubits which simply exchanges the ith and the jth qubit (for $1 \leq i < j \leq n$).

a) Use the result of the previous task to show that the action of this circuit is a unitary operation.

b) Write out explicitly the 16×16-matrix defined by this circuit for $n = 4$, $i = 2$ and $j = 4$.