

MAT 495: P.D.E Homework Problem and a Solution

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Problem: Solve the initial value problem

$$u_{tt} = 100u_{xx}, u(x, 0) = \mathcal{E}_0 \sin(kx), u_t(x, 0) = -\mathcal{E}_0 \cos(kx)$$

Solution:

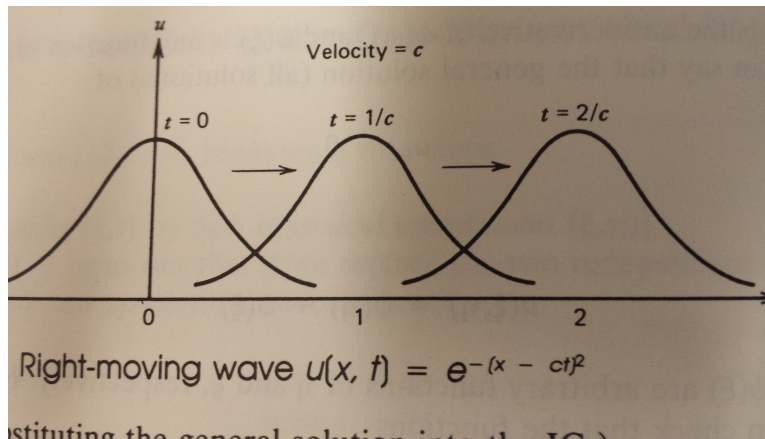
Using the *D'Alembert's solution* [1] of the wave equation in one dimension

$$u(x, t) := \frac{1}{2}[f(x-ct) + f(x+ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} g(s) ds$$

with the given initial data $f(x) = \mathcal{E}_0 \sin(kx)$, $g(x) = -\mathcal{E}_0 \cos(kx)$ and $c^2 = 100$ obtain

$$\begin{aligned} u(x, t) &:= \frac{1}{2}[\mathcal{E}_0 \sin(k(x-ct)) + \mathcal{E}_0 \sin(k(x+ct))] + \frac{1}{2c} \int_{x-ct}^{x+ct} (-\mathcal{E}_0 \cos(ks)) ds \\ &= \frac{\mathcal{E}_0}{22} [11 \sin(k(x-ct)) + 9 \sin(k(x+ct))] \end{aligned}$$

Figure 1: Right Moving Wave Plot for $e^{-(x-ct)^2}$



References

- [1] Stanley J. Farlow. Dover, New York, USA, 1993.
- [2] Bruce N. Lundberg and Aubrey B. Poore. Numerical continuation and singularity detection methods for parametric nonlinear programming. *SIAM J. Optim.*, 3(1):134–154, 1993.