

A Reading Report on: "Warning Signs of a Possible Collapse of Contemporary Mathematics"

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Math 495 Seminar Presentations

Overview

- 1 Approach to the Thought of a Technically Competent “Ultra-Finitist” in Mathematics
 - “Is infinity real? E.g. Are there infinitely many numbers? Yes indeed.”
 - Etymology of infinity: “unfinished”, always potential, never actual or completed.
 - Contrasting idea of *completed infinity* – arouses strong ‘loathing and feeling of oppression ...It is the antithesis of life, of newness, of becoming – it is finished.’
- 2 Peano Arithmetic (P)
- 3 Nelson’s Three Warning Signs

Peano Arithmetic P:

S means "Successor of"

$$① \neg Sx = 0$$

$$② Sx = Sy \rightarrow x = y$$

$$③ x + 0 = x$$

$$④ x + Sy = S(x + y)$$

$$⑤ x \cdot 0 = 0$$

$$⑥ x \cdot Sy = (x \cdot y) + x$$

$$⑦ \phi(0) \& \forall x [\phi(x) \rightarrow \phi(Sx)] \rightarrow \forall x [\phi(x)]$$

Nelson’s Three “Warning Signs”

- 1 The intuitive notion of infinite sets leads to a contradiction (e.g. Russell’s paradox).
- 2 Gödel’s Second Incompleteness Theorem: P (Peano Arithmetic) cannot be proven consistent by means expressible in P, **provided P is consistent**.
(So, ..., “perhaps P is inconsistent”)
- 3 “Most serious”: “The belief that exponentiation, superexponentiation, and so forth, applied to numerals yield numerals is just that – a belief.” (based on circular reasoning—assuming numerals exist to form them).

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Postscript

- Nelson's view of infinity only being potential is common in the history of philosophy.
- Nelson announced writing on a proof of P's inconsistency Sept. 26, 2011 which he later withdrew.

Outlook

- Need more study of Nelson's Peano Arithmetic proofs
- Need to ponder the ways he exploits exponentiation not being associative, e.g. $2^{(2^3)} \neq (2^2)^3$

For Further Reading I



Edward Nelson

“Warning Signs of a Possible Collapses of Contemporary Mathematics.”

in *Infinity: New Research Frontiers*.

ed. M. Heller and W. H. Woodin.

Cambridge University Press, 2011. p. 76-85.



Aristotle's *Physics*:

A Guided Study.

Joe Sachs

Rutgers University Press, 2001.