

FINAL EXAM PRACTICE TEST

- 1.) Compute (giving a complete, careful justification)

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-4}$$

- 2.) Prove that $f(x) = x^5 + 4x^2 - 9x + 3$ has a root (zero) in the interval $[0, 1]$.
(hint: IVT)

- 3.) a.) Where is the function $g(x) = \frac{x^2+x-2}{x+2}$ continuous?

b.) Can you extend it to another function which is continuous on a larger domain?

c.) What will be this extension?

- 4.) Compute $\lim_{h \rightarrow 0} \frac{\sqrt{1+h}-1}{h}$. Be completely specific and detailed about every single algebraic step you make and limit rule you use.

- 5.) Use numerical and graphical evidence to conjecture values for each limit.
If possible, use factoring to verify your conjecture.

a.) $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1}$

b.) $\lim_{x \rightarrow -1} \frac{x^2+x}{x^2-x-2}$

c.) $\lim_{x \rightarrow 3} \frac{3x-9}{x^2-5x+6}$

d.) $\lim_{x \rightarrow -2} \frac{2+x}{x^2+2x}$

- 6.) Sketch the graph of $f(x) = \begin{cases} x^3-1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ \sqrt{x+1}-2 & \text{if } x > 0 \end{cases}$

and identify each limit:

a.) $\lim_{x \rightarrow 0^-} f(x)$

b.) $\lim_{x \rightarrow 0^+} f(x)$

c.) $\lim_{x \rightarrow 0} f(x)$

d.) $\lim_{x \rightarrow -1} f(x)$

e.) $\lim_{x \rightarrow 1^-} f(x)$

7.) Use Numerical and graphical evidence to Conjecture whether $\lim_{x \rightarrow a} f(x)$ exists. If not, describe what is happening at $x=a$ graphically.

a.) $\lim_{x \rightarrow 0} \frac{x^2 + x}{\sin x}$

b.) $\lim_{x \rightarrow 1} \frac{\sqrt{5-x} - 2}{\sqrt{10-x} - 3}$

c.) $\lim_{x \rightarrow 0} x \csc 2x$

d.) $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$

8.) Compute $\lim_{x \rightarrow -1} \frac{x+1}{x^2+1}$, $\lim_{x \rightarrow \pi} \frac{\sin x}{x}$ and similar limits to investigate the

following. Suppose that $f(x)$ and $g(x)$ are functions with $f(a) = 0$ and $g(a) \neq 0$. What can you conjecture about $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$?

9.) Give an example of a function f such that $\lim_{x \rightarrow 0} f(x)$ exists and $f(0)$ exists, but $\lim_{x \rightarrow 0} f(x) \neq f(0)$.

10.) Evaluate the indicated limit, if it exists. Assume that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.

a.) $\lim_{x \rightarrow 0} (x^2 - 3x + 1)$

b.) $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$

c.) $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

d.) $\lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right)$

e.) $\lim_{x \rightarrow -1} f(x)$, where $f(x) = \begin{cases} 2x+1 & \text{if } x < -1 \\ 3 & \text{if } -1 < x < 1 \\ 2x+1 & \text{if } x > 1 \end{cases}$

11.) Evaluate each limit and justify each step by citing the appropriate theorem or equation.

a.) $\lim_{x \rightarrow 2} (x^2 - 3x + 1)$

b.) $\lim_{x \rightarrow 0} \frac{x-2}{x^2+1}$

12. Determine where f is continuous. If possible, extend f to a new function that is continuous on a larger domain.

a.) $f(x) = \frac{x^2 + x - 2}{x + 2}$

b.) $f(x) = \frac{x^2 - x - 6}{x - 3}$

c.) $f(x) = \begin{cases} 3x - 1 & \text{if } x \leq -1 \\ x^2 + 5x & \text{if } -1 < x < 1 \\ 3x^3 & \text{if } x \geq 1 \end{cases}$

13. Explain why the below function fails to be continuous at the given x -value by indicating which of the three conditions are not met.

(Hint: $f(a)$ must be defined)

- the limit $\lim_{x \rightarrow a}$ must exist

- the limit and the value of

f at the point must be the same.)

a.) $f(x) = \sin \frac{1}{x}$ at $x = 0$

b.) $f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 3x - 2 & \text{if } x > 2 \end{cases}$ at $x = 2$

c.) $f(x) = \frac{2x}{\sqrt{x^3 + x^2}}$ at $x = 0$

14. Use the Intermediate Value Theorem to verify that $f(x)$ has a zero in the given interval. Then use the method of bisection to find an interval of length $\frac{1}{32}$ that contains the zero.

a.) $f(x) = x^2 - 7$

a.) $[2, 3]$ a.) $[-3, -2]$

b.) $f(x) = x^3 - 4x - 2$

b.) $[2, 3]$ b.) $[-1, 0]$

15. Determine values of $a + b$ that make the given function continuous.

$$f(x) = \begin{cases} a \cos x + 1 & \text{if } x < 0 \\ \sin\left(\frac{\pi}{2}x\right) & \text{if } 0 \leq x \leq 2 \\ x^2 - x + b & \text{if } x > 2 \end{cases}$$

16. Determine a.) $\lim_{x \rightarrow a^-} f(x)$ b.) $\lim_{x \rightarrow a^+} f(x)$ and c.) $\lim_{x \rightarrow a} f(x)$

$f(x) = \frac{1 - 2x}{x^2 - 1}$ $a = -1$

17. Determine each limit with a number, ∞ , $-\infty$, or does not exist.

a.) $\lim_{x \rightarrow 2} \frac{x^2 + 2x - 1}{x^2 - 4}$

b.) $\lim_{x \rightarrow \infty} \frac{2x^2 - x + 1}{4x^2 - 3x - 1}$

c.) $\lim_{x \rightarrow \infty} \frac{3 - 2/x}{2 + 1/x}$

d.) $\lim_{x \rightarrow \infty} \frac{3x + \sin x}{4x - \cos 2x}$

18. Find a function of the form $f(x) = \frac{20x^{-0.4} + 16}{g(x)}$ such that

$\lim_{x \rightarrow 0^+} f(x) = 5$ and $\lim_{x \rightarrow \infty} f(x) = 4$.

19. Use graphical and numerical evidence to conjecture the value of the limit. Then, verify your conjecture by finding the limit exactly.

a.) $\lim_{x \rightarrow \infty} (\sqrt{4x^2 - 2x + 1} - 2x)$ (Hint: Multiply and divide by the conjugate expression: $\sqrt{4x^2 - 2x + 1} + 2x$)

b.) Suppose that $f(x)$ is a rational function $f(x) = \frac{p(x)}{g(x)}$ with the degree (largest exponent) of $p(x)$ less than the degree of $g(x)$. Determine the horizontal asymptote of $y = f(x)$.

c.) Find a quadratic function $g(x)$ such that $f(x) = \frac{x^2 - 4}{g(x)}$ has one horizontal asymptote $y = 2$ and two vertical asymptotes $x = \pm 3$.

20. Symbolically find δ in terms of ϵ .

a.) $\lim_{x \rightarrow 0} 3x = 0$

b.) $\lim_{x \rightarrow 2} (3x + 2) = 8$

c.) $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1} = 3$

21. Numerically and graphically determine a δ corresponding to (a.) $\epsilon = 0.1$ and (b.) $\epsilon = 0.05$. Graph the function in the ϵ - δ window [x-range is $(a - \delta, a + \delta)$ and y-range is $(L - \epsilon, L + \epsilon)$] to verify that your choice works.

a.) $\lim_{x \rightarrow 0} (x^2 + 1) = 1$

b.) $\lim_{x \rightarrow 1} \sqrt{x + 3} = 2$

22. Identify a specific $\Sigma > 0$ for which no $\delta > 0$ exists to satisfy the definition of limit.

a.) $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ x^2 + 3 & \text{if } x > 1 \end{cases}, \lim_{x \rightarrow 1} f(x) \neq 2$

b.) $f(x) = \begin{cases} x-1 & \text{if } x < 2 \\ x^2 & \text{if } x > 2 \end{cases}, \lim_{x \rightarrow 2} f(x) \neq 1$

23. Find the equation of the tangent line to the curve $y = \sqrt{x}$ at the point $(1, 1)$

24. Prove that $\lim_{x \rightarrow 1/2} 1-x = 1/2$, using $\delta = \Sigma$.

25. What could be the response δ to the challenge $\Sigma = .1$ in proving that $\lim_{x \rightarrow \pi/2} \cos x = 0$?

26. Find the equation of the tangent line to $y = f(x)$ at $x = a$. Graph $y = f(x)$ and the tangent line to verify that you have the correct equation.

a.) $f(x) = x^2 - 2, a = 1$ b.) $f(x) = x^2 - 3x, a = -2$ c.) $f(x) = \sqrt{x+3}, a = -2$

27. Use the position function $s(t)$ meters to find the velocity at time $t = a$ seconds.

a.) $s(t) = -4.9t^2 + 5$ 1a.) $a = 1$ 2a.) $a = 2$

b.) $s(t) = \sqrt{t+16}$ 1b.) $a = 0$ 2b.) $a = 2$

28. Use graphical and numerical evidence to explain why a tangent line to the graph of $y = f(x)$ at $x = a$ does not exist.

a.) $f(x) = |x-1|$ at $a = 1$ b.) $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$ at $a = 0$

29. Compute $f'(a)$ using the limits.

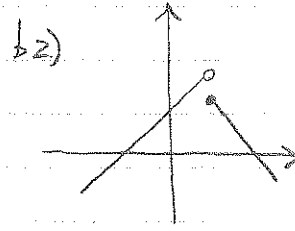
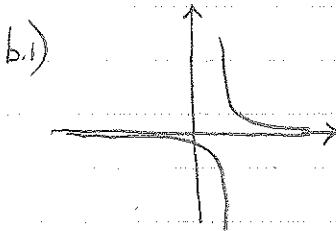
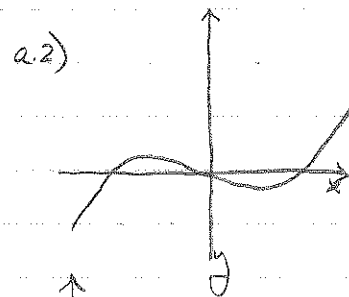
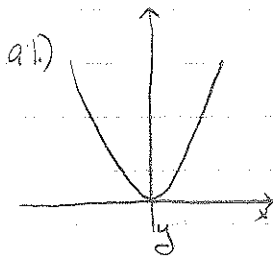
a.) $f(x) = 3x + 1, a = 1$ b.) $f(x) = \sqrt{3x+1}, a = 1$

30. Compute the derivative function f' .

a.) $f(x) = 3x^2 + 1$ b.) $f(x) = x^3 + 2x - 1$ c.) $f(x) = \frac{3}{x+1}$ d.) $f(t) = \sqrt{3t+1}$

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31. Use the graph of f to sketch a graph of f' .



32. Use the distances $f(t)$ to estimate the velocity at $t=2$.

t	1.7	1.8	1.9	2.0	2.1	2.2	2.3
$f(t)$	4.6	5.3	6.1	7.0	7.8	8.6	9.3

33. For $f(x) = \begin{cases} x^2 + 2x & \text{if } x \leq 0 \\ ax + b & \text{if } x > 0 \end{cases}$ find all numbers a and b such that $f'(0)$ exists.

34. Differentiate each function:

a.) $f(x) = x^3 - 2x + 1$

b.) $f(s) = 5\sqrt{s} - 4s^2 + 3$

c.) $h(x) = \frac{10}{3\sqrt{x}} - 2x + \pi$

d.) $f(s) = 2s^{3/2} - 3s^{-1/3}$

e.) $f(x) = \frac{3x^2 - 3x + 1}{2x}$

f.) $f(x) = x(3x^2 - \sqrt{x})$

35. Compute the indicated derivative.

a.) $f''(t)$ for $f(t) = t^4 + 3t^2 - 2$

b.) $\frac{d^2 f}{dx^2}$ for $f(x) = x^6 - \sqrt{x}$

36. Find an equation of the tangent line to $y = f(x)$ at $x = a$

a.) $f(x) = x^2 - 2$, $a = 2$

b.) $f(x) = x^2 - 2x + 1$, $a = 2$

c.) $f(x) = 4\sqrt{x} - 2x$, $a = 1$

37. Find the derivative of each function.

a.) $f(x) = (x^2 + 3)(x^3 - 3x + 1)$

b.) $f(x) = (x^{3/2} - 4x)(x^4 - \frac{3}{x^2} + 2)$

c.) $g(t) = \frac{3t - 2}{5t + 1}$

d.) $f(x) = \frac{3x - 6\sqrt{x}}{5x^2 - 2}$

e.) $f(u) = \frac{(u+1)(u-2)}{u^2 - 5u + 1}$

f.) $f(x) = \frac{x^2 + 3x - 2}{\sqrt{x}}$

38. Find an equation of the tangent line to the graph of $y = f(x)$ at $x = a$.

a.) $f(x) = (x^2 + 2x)(x^4 + x^3 + 1)$, $a = 0$ b.) $f(x) = \frac{x+1}{x+2}$, $a = 0$

39. Assume that f and g are differentiable with $f(0) = -1$, $f(1) = -2$, $f'(0) = -1$, $f'(1) = 3$, $g(0) = 3$, $g(1) = 1$, $g'(0) = -1$, $g'(1) = -2$. Find an equation of the tangent line to the graph of $y = h(x)$ at $x = a$.

a.) $h(x) = f(x)g(x)$ at (a) $a = 0$ (b) $a = 1$ b.) $h(x) = \frac{f(x)}{g(x)}$ at (a) $a = 1$, (b) $a = 0$

EXAM

40. Make a graph and label it carefully and thoroughly to help you find a number $\delta > 0$ such that $|\sqrt{4x+1} - 3| < 0.5$ whenever $|x - 2| < \delta$. That is, find the (numerical) value of δ which is your response to the challenge $\Sigma = .5$ in collecting evidence for $\lim_{x \rightarrow 2} \sqrt{4x+1} = 3$.

41. How do we know that $2x^3 + x^2 + 2 = 0$ has a root between -2 and -1 ?

42. Prove, using the $\Sigma - \delta$ definition of the limit, that $\lim_{x \rightarrow 2} (14 - 5x) = 4$.

43. Carefully compute $\lim_{x \rightarrow \infty} (\sqrt{5x^2 + 4x + 7} - \sqrt{5x^2 + x - 3})$.

44. Find the derivative with and without using the chain rule.

a.) $f(x) = (x^3 - 1)^2$

b.) $f(x) = (x^2 + 1)^3$

45. Differentiate each function.

a.) $f(x) = (x^3 - x)^3$

b.) $f(x) = \sqrt{x^2 + 4}$

c.) $f(t) = t^5 \sqrt{t^3 + 2}$

d.) $f(t) = (t^3 + 2)\sqrt{t}$

e.) $f(u) = \frac{u^2 + 1}{u + 4}$

f.) $g(x) = \frac{x}{\sqrt{x^2 + 1}}$

g.) $h(x) = \frac{6}{\sqrt{x^2 + 4}}$

h.) $f(x) = (\sqrt{x^3 + 2} + 2x)^{-2}$

46. Find the derivative, where f is an unspecified differentiable function.

a.) $f(x^2)$

b.) $[f(x)]^2$

c.) $f(\frac{1}{x})$

d.) $\frac{1}{f(x)}$

e.) $1 + f(x^2)$

47. Find the second derivative.

a.) $f(x) = \sqrt{x^2 + 4}$

b.) $f(t) = \frac{2}{\sqrt{t^2 + 4}}$

(quiz) 48. Let $y = \sqrt{3+5\sqrt{x^3+\pi}}$ Find $\frac{dy}{dx}$

49. Find the derivative of each function.

a) $f(x) = 4\sin 3x - x$ b.) $f(x) = x \cos 5x^2$ c.) $f(x) = \frac{\sin x^2}{x^2}$

d.) $f(x) = \frac{1}{\sin 4x}$ e.) $f(x) = \tan \sqrt{x^2+1}$ f.) $f(x) = \sin^3(\cos \sqrt{x^3+2x^2})$

50. Differentiate each function.

a.) $f(x) = \sin x^2 \tan x$ b.) $f(x) = \sin^2(\tan x)$ c.) $f(x) = \sin(\tan^2 x)$

51. Find an equation of the tangent line to $y = f(x)$ at $x = a$.

a.) $f(x) = \sin 4x, a = \frac{\pi}{8}$ b.) $f(x) = x^2 \cos x, a = \frac{\pi}{2}$

52. Compute the slope of the tangent line at the given point both explicitly and implicitly.

a.) $x^2 + 4y^2 = 8$ at $(2, 1)$ b.) $y - 3x^2y = \cos x$ at $(0, 1)$

53. Find the derivative $y'(x)$ implicitly.

a.) $x^2y^2 + 3y = 4x$ b.) $\frac{x+3}{y} = 4x + y^2$ c.) $y^2\sqrt{x+y} - 4x^2 = y$ d.) $\tan(y^2+3) - xy^2 = 2x$

54. Find the locations of all horizontal and vertical tangents.

a.) $x^2 + y^3 - 3y = 4$ b.) $xy^2 - 2y = 2$

quiz 5 55. What is the value of $\lim_{x \rightarrow 0} \frac{\tan x}{x}$?

56. Find the equation of the tangent line to the curve $x^2 + y^3 - 2y = 3$, at the point $(2, 1)$

57. Explain how we can be completely sure that the equation $x^3 + 4x - 3$ has one solution.

58. Explain Rolle's and the Mean Value Theorems.

59. Check the hypotheses of Rolle's Theorem and The Mean Value Theorem and find a value of c that makes the appropriate conclusion true, and graph.

a.) $f(x) = x^2 + 1, [-2, 2]$

b.) $f(x) = \sin x, [0, \pi/2]$

60. Find all functions g such that $g'(x) = f(x)$.

a.) $f(x) = x^2$

b.) $f(x) = 1/x^2$

c.) $f(x) = \sin x$

d.) $f(x) = \frac{\sin x}{\cos^2 x}$

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quiz 6

61.

62.

63.

64. Find the linear approximation to $f(x)$ at $x = x_0$. Use the linear approx. to estimate the given number.

a.) $f(x) = \sqrt{x}, x_0 = 1, \sqrt{1.2}$

b.) $f(x) = \sqrt{2x+9}, x_0 = 0, \sqrt{8.8}$

65. Find the linear approx. at $x=0$ to show that the following commonly used approx. are valid for "small" x . Compare the approx. and exact values for $x = 0.01$, $x = 0.1$, and $x = 1$.

a.) $\sqrt{1+x} \approx 1 + \frac{1}{2}x$

66. Find the critical numbers by hand. Determine if local maximum, local minimum or neither.

a.) $f(x) = x^2 + 5x - 1$

b.) $f(x) = x^3 - 3x^2 + 6x$

c.) $-x^3 + 3x^2 - 3x$

d.) $f(x) = x^4 - 2x^2 + 1$

67. Find the absolute extrema of the given function on each indicated interval.

a.) $f(x) = x^3 - 3x + 1$ on (a.) $[0, 2]$ and (b.) $[-3, 2]$

b.) $f(x) = \frac{3x^2}{x-3}$ on (a.) $[-2, 2]$ and (b.) $[2, 8]$

quiz 68. Find the point (x, y) on the parabola $y = 9 - x^2$ which is closest to the point $(3, 9)$. Also, what is that minimum distance?

69. Find, by hand, the intervals where the function is increasing and decreasing. Determine all local extrema and sketch a graph.

a) $y = x^3 - 3x + 2$ b.) $y = x^4 - 8x^2 + 1$ c.) $y = (x+1)^{2/3}$ d.) $y = \sin x + \cos x$

70. Find all crit. #s and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

a.) $y = x^2 - 2x^{2/3} + 2$ b.) $y = \frac{x}{1+x^3}$

71. Sketch a graph of a function with the given properties.

a.) $f(0)=1$, $f(2)=5$, $f'(x) < 0$ for $x < 0$ and $x > 2$, $f'(x) > 0$ for $0 < x < 2$.

72. Estimate critical numbers and sketch graphs showing both global and local behavior.

a.) $y = \frac{x+60}{x^2+1}$

73. Determine the intervals where the graph of the given function is concave up and concave down and identify inflection points.

a.) $f(x) = x^3 - 3x^2 + 4x - 1$ b.) $f(x) = x^{4/3} + 4x^{1/3}$ c.) $f(x) = x + 3(1-x)^{1/3}$

74. Find all crit. #s and use the Second Derivative Test to determine all local extrema.

a.) $f(x) = x^4 + 4x^3 - 1$ b.) $f(x) = \frac{x^2 - 5x + 4}{x}$ c.) $f(x) = \frac{x^2 - 1}{x}$

75. Determine all significant features (approx. if necessary) and sketch a graph.

a.) $f(x) = x^4 - 26x^3 + x$ b.) $f(x) = x^4 + 32x^3 - 0.02x^2 - 0.8x$

76. Graph the function and determine the critical numbers, inflection points, local max-min, absolute max-min and domain.

a.) $f(x) = x^3 - 3x^2 + 3x$ b.) $f(x) = x^4 - 3x^2 + 2x$

c.) $f(x) = x^6 - 10x^5 - 7x^4 + 80x^2 + 12x^2 - 192x$

d.) $f(x) = x^4 + 32x^3 - 0.02x^2 - 0.8x$

77. A 3-sided fence is to be built next to a straight section of river, which forms the fourth side of a rectangular region. There is 96 feet of fencing available. Find the maximum enclosed area and the dimensions of the corresponding enclosure.
78. A 2-pen Corral is to be built. The outline of the Corral forms two identical adjoining rectangles. If there is 120 ft. of fencing available, what dimensions of the Corral will maximize the enclosed area?
79. A box with no top is to be built by taking a 12"-by-16" sheet of cardboard and cutting x -in. squares out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box.
80. Find the point on the curve $y = x^2$ closest to the point (3,4).
81. A company needs to run an oil pipeline from an oil rig 25 miles out to sea to a storage tank that is 5 miles inland. The shoreline runs east-west and the tank is 8 miles east of the rig. Assume it costs \$50 thousand per mile to construct the pipeline on land. The pipeline will be built in a straight line from the rig to a selected point on the shoreline, then in a straight line to the storage tank. What point on the shoreline should be selected to minimize the total cost of the pipeline?
82. Suppose a wire 2 ft. long is to be cut into two pieces, each of which will be formed into a square. Find the size of each piece to maximize the total area of the two squares.

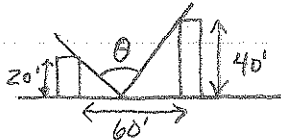
83. Suppose that group tickets to a concert are priced at \$40 per ticket if 20 tickets are ordered, but cost \$1 per ticket less for each extra ticket ordered, up to a maximum of 50 tickets.

a.) Find the # of tickets

b.) If management wanted the solution to part a.) to be 50, how much should the price be discounted for extra tickets ordered?

84. Oil spills out of a tanker at the rate of 90 gallons per minute. The oil spreads in a circle with a thickness of $\frac{1}{8}$ ". Determine the rate at which the radius of the spill is increasing when the radius reaches 100 ft.

85. Two buildings of height 20 feet and 40 feet, respectively, are 60 feet apart. Suppose that the intensity of light at a point between the buildings is proportional to the angle θ in the figure. (a.) If a person is moving from right to left at 4 ft/s, at what rate is θ changing when the person is exactly halfway between the two buildings? (b.) Find the location at which the angle θ is maximum.



EXAM II. 86. A balloon is rising straight up at a constant speed of 5 ft/sec. A boy is cycling along a straight and level road at a speed of 15 ft/sec. The boy passes under the balloon at the moment it is already 45 ft above his head. How fast is the distance between the boy and balloon increasing 3 sec. later?

87. Find $f'(x)$ for the function $f(x) = \sin^3(\cos \sqrt{x^3 + 2x^2})$

88. Verify that the function $f(x) = \sqrt[3]{x}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 1]$. Then find all the numbers c that satisfy the conclusion of the MVT.

89. A farmer wants to fence off a rectangular field of area 1.5 Million square feet, and then divide it in half with a fence running down the middle, parallel to one of the sides of the rectangle. How can she do this so as to minimize the length of fencing it will require?

90. Sketch several members of the family of functions defined by the antiderivative.

a.) $\int x^3 dx$ b.) $\int (x^3 - x) dx$ c.) $\int (x-2) dx$

91. Find the anti-derivative.

a.) $\int (3x^4 - 3x) dx$ b.) $\int (x^3 - 2) dx$ c.) $\int (3\sqrt{x} - \frac{1}{x^4}) dx$ d.) $\int (2x^{-2} + \frac{1}{\sqrt{x}}) dx$

e.) $\int \frac{x^{1/3} - 3}{x^{2/3}} dx$ f.) $\int \frac{x + 2x^{3/4}}{x^{5/4}} dx$ g.) $\int (5x - \frac{3}{x^2}) dx$ h.) $\int x^{2/3} (x^{-4/3} - 3) dx$

92. Find the function $f(x)$ satisfying the given conditions.

a.) $f'(x) = x^2 + x$, $f(0) = 4$

b.) $f''(x) = 20x^3 + 2x$, $f'(0) = -3$, $f(0) = 2$

93. Determine the position function if the velocity function is $v(t) = 3\cos t - 2$ and the initial position is $s(0) = 0$.

94. Translate into summation notation.

a.) $2(1)^2 + 2(2)^2 + 2(3)^2 + \dots + 2(14)^2$

b.) $\sqrt{2-1} + \sqrt{3-1} + \sqrt{4-1} + \dots + \sqrt{15-1}$

95. Write out the terms and compute the sums.

a.) $\sum_{i=1}^6 3i^2$

b.) $\sum_{i=3}^7 (i^2 + i)$

c.) $\sum_{i=6}^{10} (4i + 2)$

d.) $\sum_{i=6}^8 (i^2 + 2)$

96. Use summation rules to compute the sum.

a.) $\sum_{i=1}^{70} (3i - 1)$

b.) $\sum_{i=1}^{40} (4 - i^2)$

c.) $\sum_{i=1}^{50} (8 - i)$

d.) $\sum_{n=1}^{100} (n^2 - 3n + 2)$

97. List the evaluation points corresponding to the midpoint of each subinterval, sketch the function and approximating rectangles and evaluate the Riemann sum.

a.) $f(x) = x^2 + 1$, (a.) $[0, 1]$, $n = 4$ (a.2) $[0, 2]$, $n = 4$

b.) $f(x) = 4 - x^2$, (a.) $[-1, 1]$, $n = 4$ (b.2) $[-3, 1]$, $n = 4$

quiz 8 98. Find the function $f(\theta)$ which satisfied

$$f''(\theta) = \sin \theta + \cos \theta, \quad f(0) = 3, \quad f'(0) = 4$$

99. Use the Midpoint Rule with $n=6$ to estimate the value of the integral

a.) $\int_0^3 (x^3 + x) dx$

b.) $\int_{-2}^2 \sqrt{4-x^2} dx$

100. Give an area interpretation of the integral.

$$\int_0^2 (x^3 - 3x^2 + 2x) dx$$

101. Compute $\int_0^4 f(x) dx$.

$$f(x) = \begin{cases} 2 & \text{if } x \leq 2 \\ 3x & \text{if } x > 2 \end{cases}$$

102. Use Theorem (If f and g are integrable on $[a, b]$, then the following are true: a.) For any constants c and d , $\int_a^b [cf(x) + dg(x)] dx = c \int_a^b f(x) dx + d \int_a^b g(x) dx$ and b.) For any c in $[a, b]$, $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$.) to write the expression as a single integral.

(a.) $\int_0^2 f(x) dx + \int_2^3 f(x) dx$

(b.) $\int_{-1}^2 f(x) dx + \int_2^3 f(x) dx$

quiz 9 103. Compute $\int_1^9 \frac{x-1}{\sqrt{x}} dx$

104. What is $\frac{dy}{dx}$ if $y = \int_0^x \frac{\cos \theta}{\theta} d\theta$?

105. Use Part I of the Fundamental Theorem to compute each integral exactly.

a.) $\int_0^2 (2x-3) dx$

b.) $\int_0^3 (x^2-2) dx$

c.) $\int_{-1}^1 (x^3+2x) dx$

d.) $\int_1^2 \frac{x^2-3x+4}{x^4} dx$

106. Find the derivative $f'(x)$.

a.) $f(x) = \int_0^x (t^2 - 3t + 2) dt$

b.) $f(x) = \int_2^x (t^2 - 3t) dt$

107. Find the position function $s(t)$ from the given velocity or acceleration function and initial values. Assume that units are feet and seconds.

a.) $v(t) = 40 - \sin t, s(0) = 2$

b.) $v(t) = 10 - t^2, s(0) = 2$

108. Find an equation of the tangent line at the given value of x .

a.) $y = \int_0^x \sin \sqrt{t^2 + \pi^2} dt$, $x = 0$ b.) $y = \int_{-1}^x \sqrt{t^2 + 2t + 2} dt$, $x = -1$

Exam 3

109. A group of MIT students wanted to measure how the Green Building on their campus is (it's actually gray; it's the only very tall building on the MIT campus.) So, naturally, they did it by gently pushing a piano off the edge of the building's roof and timing how long it took to hit the ground. They found it took 6.32456 seconds. How tall is the Green Building? Also, how fast was the piano moving when it hit? (The only force on piano is gravity)

110. Compute an approximation to the area under the curve $y = \sqrt{x}$ from $x = 0$ to $x = 2$ by using a Riemann sum w/ $n = 4$ terms and taking as your sample point the right endpoints. Draw a picture of what this is doing - is your number an over-estimate or under-estimate of the actual area $\int_0^2 \sqrt{x} dx$?

111. Use some basic integration rules and then geometry to evaluate $\int_{-1}^0 (x + \sqrt{1-x^2}) dx$

112. Compute $\frac{dy}{dx}$ if $y = \int_{x^2}^{17}$. Name all theorems used.

113. What is the most general antiderivative $\int \frac{x}{\sqrt{x+2}} dx$. Show all work.

114. Compute $\int_0^1 x^2 (1+2x^3)^5 dx$.

115. Find the geometrical area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \frac{\pi}{2}$.

116. Compute the volume of the foot ball-shaped three-dimensional shape formed by rotating the region bounded by the curves $x = y - y^2$ and $x = 0$ around the y -axis.

117. Use the given substitution to evaluate the indicated integral.

a.) $\int x^2 \sqrt{x^3+2} \, dx, u = x^3+2$ b.) $\int \sin x \cos x \, dx, u = \sin x$

118. Evaluate the indicated integral.

a.) $\int x^3 \sqrt{x^4+3} \, dx$ b.) $\int \sqrt{1+10x} \, dx$ c.) $\int \frac{v}{\sqrt{v^2+4}} \, dv$ d.) $\int \frac{x^3}{\sqrt{1-x^4}} \, dx$

119. Make the indicated substitution for an unspecified function $f(x)$.

$u = x^3$ for $\int_1^2 x^2 f(x^3) \, dx$

120. Find the area between the curves on the given interval.

a.) $y = x^3, y = x^2 - 1, 1 \leq x \leq 3$ b.) $y = x^4, y = x - 1, -2 \leq x \leq 0$

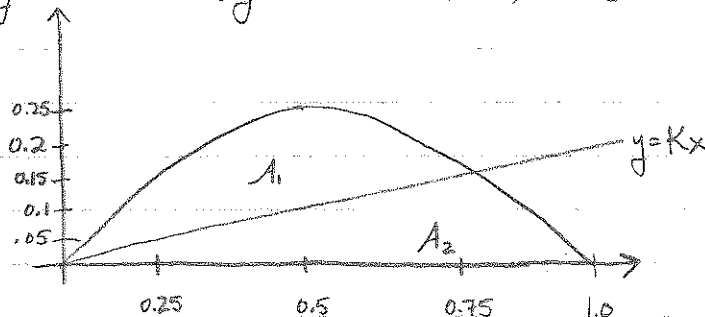
121. Sketch and find the area of the region determined by the intersections of the curves.

a.) $y = x^2 - 1, y = 7 - x^2$ b.) $y = \sqrt{x}, y = x^2$

122. Sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral.

a.) $y = x, y = 2 - x, y = 0$ b.) $x = 3y, x = 2 + y^2$

123. For $y = x - x^2$ and $y = Kx$ as shown, find K such that $A_1 = A_2$



124. Find the volume of the solid with cross-sectional area $A(x)$.

a.) $A(x) = x + 2, -1 \leq x \leq 3$
 b.) $A(x) = 10 + 6x^2, 0 \leq x \leq 10$
 c.) $A(x) = 2(x+1)^2, 1 \leq x \leq 4$

125. Find the volume of a pyramid of height 160 ft. that has a square base of side 300 ft. How does the volume compare?
126. A pottery jar has circular cross sections of radius $4 - \sin \frac{x}{2}$ inches for $0 \leq x \leq 2\pi$. Sketch the jar and compute its volume.
127. Compute the volume of the solid formed by revolving the given region about the given line.
 - Region bounded by $y = x^2$ and $x = y^2$ about (a) the y -axis;
 (b) $x = 1$.
128. Let R be the region bounded by $y = x$, $y = -x$ and $x = 1$. Compute the volume of the solid formed by revolving R about the given line.
 a.) the x -axis (b.) the y -axis (c.) $y = 1$ (d.) $y = -1$
129. Sketch the region, draw in a typical shell, identify the radius and height of each shell and compute the volume.
 a.) The region bounded by $y = x^2$ and the x -axis, $-1 \leq x \leq 1$, revolved about $x = 2$.
 b.) The region bounded by $y = x^2$ and the x -axis, $-1 \leq x \leq 1$, revolved about $x = -2$.
 c.) The region bounded by $y = x$, $y = -x$ and $x = 1$ revolved about $x = 1$.
130. The region bounded by $y = x^2$ and $y = 2 - x^2$, revolved about $x = 2$.
131. The region bounded by $x = y^2$ and $x = 4$ revolved about $y = 2$.
132. The integral represents the volume of a solid. Sketch the region and axis of revolution that produces the solid.
 a.) $\int_0^1 \pi [(\sqrt{y})^2 - y^2] dy$ b.) $\int_0^2 \pi (4 - y^2)^2 dy$

133. A weight lifter lifts 250 pounds a distance of 20 inches. Find the work done in stretching this spring 3 inches beyond its natural length.
134. A wrestler lifts his 300-pound opponent over head, a height of 6 feet. Find the work done (as measured in foot-pounds.)
135. A 40-foot chain weighs 1000 pounds and is hauled up to the deck of a boat. The chain is oriented vertically and the top of the chain starts 30 ft. below the deck. Compute the work done.
136. A right circular cylinder of radius 1 m and height 3 m is filled with water. Compute the work done pumping all the water out the top of the cylinder if (a) the cylinder stands upright (the circular cross sections are parallel to the ground.) and (b) the cylinder cross sections are perp. to the ground.)