

MATH 156.6 — TEST III SOLUTIONS

1

Let X be the RV “dollars paid per month for Internet access”, where the OUs are “households in the US.” For an SRS of size 500, let $\bar{X}$ be, as usual, the sample mean of X . The Central Limit Theorem (CLT) says that $\bar{X}$ will be approximately Normal with mean equal to the population mean of X (which the problem tells us is \$38) and standard deviation equal to $\sigma/\sqrt{500}$, where σ is the population standard deviation of X and the 500 is the sample size. That is, $\bar{X}$ is $N(38, 10/\sqrt{500}) = N(38, .4472)$. Hence “the probability that the sample mean will be more than \$39” is $P(\bar{X} > 39) = \text{normalcdf}(39, 9999, 38, .4472) = .01267$. The CLT results are quite accurate, essentially no matter how non-Normal the original data was, since the sample size is quite large.

2

(a) Here is a stemplot of this data:

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      4  0
      3  0  3  4  5  5
      2  2  2  3  3  6  7  8  9
      1  1  2  4  8
    
```

This data is not perfectly symmetric or normal, certainly, but it seems to have only one main peak and no obvious outliers, so we can be reasonably content using it to make a confidence interval. Note that it is even more important, in terms of data quality, that the data should have come from an SRS. However, that is something we must merely assume, and we cannot in any way verify it by looking at the data. A histogram or box-and-whisker plot would also give similar information.

(b)

Let X be the RV “wound healing rate for a newt” where the OUs are newts from the world population of the species the problem is talking about. Also let μ be the population mean of X . Repeating: **we must assume that the data in the problem comes from an SRS** of this species of newts in order for our CIs to be at all accurate. Compute a 90% confidence interval for μ using the formula

$$\bar{X} \pm z^* \frac{\sigma}{\sqrt{n}} = 25.66 \pm 1.645 \frac{8}{\sqrt{18}} = 25.66 \pm 3.11$$

where here

the sample mean is $\bar{X} = 25.66$ (computed from the given data),
the sample size is $n = 18$ (value from counting the given data values),
the critical value $z^* = 1.645$ for a confidence level of 90% comes from the table in the test,

and

the population standard deviation of X is $\sigma = 8$ (given in the problem).

Thus the desired 90% confidence interval is

(22.565, 28.768) [interval notation].

- (c) “90% confidence” means:

If we were to collect many more SRS of the same size as before and recalculate new sample means $\bar{X}$ and then new CIs from these samples, we would expect that at least 90% of these new CIs would contain the one, fixed, unknown value of μ (the true population mean of the would healing rate for all newts of this species).

- (d) The margin of error is $z^* \frac{\sigma}{\sqrt{n}}$, where these variables are defined as above. If we want to change the sample size and end up with a margin of error of 1, keeping the population standard deviation σ and the confidence level (so also the z^*) unchanged, we must therefore solve

$$1 = 1.645 \frac{8}{\sqrt{n}}$$

for n . But then

$$n = (1.645 \cdot 8)^2 = 173.18.$$

n must be a whole number so

for any whole number $n \geq 174$ we get a margin of error of less than $1\mu m/hr$.

3

- (a) Let

X be the RV “SAT score”

for

the OUs “SAT-taking students”

out of

the population of those who have or who ever will have completed the *JP’s MTS* study course.

Let

μ be the population average of X for all such students.

JP’s advertisement is the claim that $\mu > 475$, while the prosecutor wants to prove instead that $\mu = 475$, where the 475 here is the SAT average across the whole country. So we are testing the null and alternative hypotheses

$$H_0 : \mu = 475$$

$$H_a : \mu > 475$$

The next step in this hypothesis test is to compute the test statistic, which is a z -statistic since we are given

the population standard deviation $\sigma = 100$.

Thus

$$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{478 - 475}{100/\sqrt{1000}} = .948$$

Note that we have to assume that the $n = 1000$ students for whom we have records are essentially an SRS out of the population in this test (not a good assumption!). We are given in the problem statement the value of

the sample mean $\bar{X} = 478$ of X .

The third step is to compute the p -value, which is [one-sided Z -test]

$$p = P(Z > z) = \text{normalcdf}(.948, 9999) = .172$$

The last step is to interpret this p -value: since p is greater than any usual level (such as .01 or .05)

We fail to reject the null hypothesis.

or, in this situation

There is insufficient statistical evidence to support the claim of this advertisement.

- (b) Just a few small differences with (a):

Let

X be the RV “SAT score”

for

the OUs “SAT-taking students”

out of

the population of those who have or who ever will have read the book *JP's T&T*.

Let

μ be the population average of X for all such students.

JP's advertisement is the claim that $\mu > 475$, while the prosecutor wants to prove that $\mu = 475$, where the 475 here is the SAT average across the whole country. So we are testing the null and alternative hypotheses

$$H_0 : \mu = 475$$

$$H_a : \mu > 475$$

Next we compute the z -statistic, using

the population standard deviation $\sigma = 100$.

Thus

$$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{478 - 475}{100/\sqrt{10000}} = 3$$

We must assume that the $n = 10000$ students who read *JP's T&T* for whom we have records are an SRS of the population in this test (not a good assumption!) – from which sample we have (from the problem statement)

the sample mean of $\bar{X} = 478$.

Third step: compute the p -value, which is (again a one-sided Z -test)

$$p = P(Z > z) = \text{normalcdf}(3, 9999) = .001$$

Interpret this: now p is less than any standard value (such as .01 or .05), so

We reject the null hypothesis.

or

There is good statistical evidence to support the claim of this advertisement.

(c) The p -value is:

The probability that we see data as extreme as what we actually saw, or even more extreme, given that H_0 is true.

Since we did actually see that data, if the p -value is quite small (the usual cut-offs are .01 or .05; these are values accepted in court) then we think it is unlikely that H_0 could have been true, so instead “we reject H_0 .”

4

(a) Make a stemplot or histogram of this data [a test solution must have this actual graphic], you will see it is fairly skewed, and either has some serious outliers or has two separate peaks. Hence it is not particularly good for CIs, particularly since the sample size is so small. Of course, we also do not know that the data is from an SRS, which is the most important condition we should require.

(b) Let

X be the RV “percentage of nitrogen in small volumes of air from the late Cretaceous”

where

the OUs are all small volumes of Cretaceous air.

Also let

μ be the population mean of X , so the mean percentage of nitrogen in Cretaceous air.

We must assume that the 9 bubbles of air from these pieces of amber are an SRS of small volumes of actual Cretaceous air, for our CIs to be at all accurate.

Since we do not know the population standard deviation of X , we compute a 95% confidence interval for μ using the formula

$$\bar{X} \pm t^* \frac{S_X}{\sqrt{n}} = 59.588 \pm 2.306 \frac{6.255}{\sqrt{9}} = 59.588 \pm 4.808$$

where here

the sample mean is $\bar{X} = 59.588$ (computed from the given data),

the sample size is $n = 9$ (value from counting the given data values),

the T-distr. crit. val. $t^* = 2.306$ for confidence level 90%, from the table in the test,

and

the sample standard deviation of X is $S_X = 8$ (from given data with `1 var stats` on calc.).

Thus the desired 95% confidence interval is

(54.781, 64.397) [interval notation].

- 5** (a) New row is the difference of the previous rows: so

1 -10 -3 3 31 -4 12 3 7 10 22 4 1 3

The reason we do this is that we can then track the improvement (perhaps) in each individual frog's rate of healing with the electric field.

Make a stemplot, histogram or boxplot and perform an analysis like in **2(a)** or **4(a)** and you will conclude that the data quality seems quit reasonable to make a CI here [test solution must have the graphic here and discuss its features – peaks, shape, skew, outliers...].

- (b) This is like problem **3**, except, since we do not know the population standard deviation, we use a T -test. So let

X be the RV “difference in rate of healing in electrified *vs* non-electrified frogs”

for

the OUs “frogs”

out of

the population of all frogs on the planet.

Let

μ be the population average of X for all frogs.

Question is if treatment *changes* the rate of healing. So use

$$H_0 : \mu = 0$$

$$H_a : \mu > 0$$

(since μ represents the difference in healing rates, being 0 corresponds to no change, and being nonzero corresponds to some change). Next compute the test statistic, now a t -statistic:

$$t = \frac{\bar{X} - \mu_0}{S_X / \sqrt{n}} = \frac{5.71 - 0}{10.56 / \sqrt{14}} = 2.02$$

We must assume that the frogs for which we have data are an SRS of size $n = 14$ out of all of the frogs on Earth, which sample has a

sample mean $\bar{X} = 5.71$ of X .

Third, compute the p -value (two-sided test!):

$$p = 2 \cdot P(T > |t|) = 2\text{tcdf}(2.02, 9999, 13) = .064$$

Interpret the p -value:

at both the significance levels of 1% and 5%, we fail to reject the null hypothesis.

Or,

There is insufficient statistical evidence to conclude, at the 1% or 5% significance level, that electrification changes the rate of healing of frog's limbs.